

# Second harmonic generation with full Poincaré beams

LI ZHANG,<sup>1,2</sup> XIAODONG QIU,<sup>2</sup> FANGSHU LI,<sup>2</sup> HAIGANG LIU,<sup>3</sup> XIANFENG CHEN,<sup>3,4</sup> AND LIXIANG CHEN<sup>2,5</sup>

<sup>1</sup>School of Physics and Optoelectronic Engineering, Foshan University, Foshan 52800, China
<sup>2</sup>Department of Physics, Jiujiang Research Institute and Collaborative Innovation Center for
Optoelectronic Semiconductors and Efficient Devices, Xiamen University, Xiamen 361005, China
<sup>3</sup>State Key Laboratory of Advanced Optical Communication Systems and Networks, School of Physics
and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China
<sup>4</sup>xfchen@sjtu.edu.cn
<sup>5</sup>chenlx@xmu.edu.cn

**Abstract:** We report a concise yet efficient experiment to extend the study of full Poincaré beams to incorporate the nonlinear optical effect. The main feature of our scheme is the employment of Type-II phase-matching KTP crystal to implement the second harmonic generation with structured vector light from invisible to visible region. Of particular interest is the revelation and visualization of the hidden topological structures transferred from the input polarization state to the output observable intensity patterns. The experimental results are in good agreement with the numerical simulations. Our present work provides us with the insight into the interaction of full Poincaré beams with media in the nonlinear regime.

© 2018 Optical Society of America under the terms of the OSA Open Access Publishing Agreement

OCIS codes: (190.2620) Harmonic generation and mixing; (260.5430) Polarization; (050.4865) Optical vortices.

#### **References and links**

- Q. Zhan, "Cylindrical vector beams: from mathematical concepts to applications," Adv. Opt. Photonics 1(1), 1– 57 (2009).
- J. F. Nye, "Lines of circular polarization in electromagnetic wave fields," Proc. R. Soc. Lond. A Math. Phys. Sci. 389 (1797), 279–290 (1983).
- M. R. Dennis, "Polarization singularities in paraxial vector fields: morphology and statistics," Opt. Commun. 213(4), 201–221 (2002).
- R. Dorn, S. Quabis, and G. Leuchs, "Sharper focus for a radially polarized light beam," Phys. Rev. Lett. 91(23), 233901 (2003).
- W. Zhu, V. Shvedov, W. She, and W. Krolikowski, "Transverse spin angular momentum of tightly focused full Poincaré beams," Opt. Express 23(26), 34029–34041 (2015).
- K. Venkatakrishnan and B. Tan, "Interconnect microvia drilling with a radially polarized laser beam," J. Micromech. Microeng. 16(12), 2603–2607 (2006).
- A. E. Willner, H. Huang, Y. Yan, Y. Ren, N. Ahmed, G. Xie, C. Bao, L. Li, Y. Cao, Z. Zhao, J. Wang, M. P. J. Lavery, M. Tur, S. Ramachandran, A. F. Molisch, N. Ashrafi, and S. Ashrafi, "Optical communications using orbital angular momentum beams," Adv. Opt. Photonics 7(1), 66–106 (2015).
- Q. Zhan, "Trapping metallic Rayleigh particles with radial polarization," Opt. Express 12(15), 3377–3382 (2004).
- L. G. Wang, "Optical forces on submicron particles induced by full Poincaré beams," Opt. Express 20(19), 20814–20826 (2012).
- A. M. Beckley, T. G. Brown, and M. A. Alonso, "Full Poincaré beams," Opt. Express 18(10), 10777–10785 (2010).
- A. M. Beckley, T. G. Brown, and M. A. Alonso, "Full Poincaré beams II: partial polarization," Opt. Express 20(9), 9357–9362 (2012).
- G. Milione and R. R. Alfano, "Cylindrical vector beam transformations and hybrid vector beams," in *Frontiers in Optics 2010/Laser Science XXVI*, OSA Technical Digest (CD) (Optical Society of America, 2010), paper FWC4.
- G. M. Lerman, L. Stern, and U. Levy, "Generation and tight focusing of hybridly polarized vector beams," Opt. Express 18(26), 27650–27657 (2010).
- H.-T. Wang, X.-L. Wang, Y. Li, J. Chen, C.-S. Guo, and J. Ding, "A new type of vector fields with hybrid states of polarization," Opt. Express 18(10), 10786–10795 (2010).
- F. Cardano, E. Karimi, L. Marrucci, C. de Lisio, and E. Santamato, "Generation and dynamics of optical beams with polarization singularities," Opt. Express 21(7), 8815–8820 (2013).

#320895 Journal © 2018 https://doi.org/10.1364/OE.26.011678

#### Research Article

#### Optics EXPRESS

- 16. X. Lu, Z. Wu, W. Zhang, and L. Chen, "Polarization singularities and orbital angular momentum sidebands from rotational symmetry broken by the Pockels effect," Sci. Rep. **4**(1), 4865 (2014).
- F. Flossmann, U. T. Schwarz, M. Maier, and M. R. Dennis, "Polarization singularities from unfolding an optical vortex through a birefringent crystal," Phys. Rev. Lett. 95(25), 253901 (2005).
- A. Desyatnikov, T. A. Fadeyeva, V. G. Shvedov, Y. V. Izdebskaya, A. V. Volyar, E. Brasselet, D. N. Neshev, W. Krolikowski, and Y. S. Kivshar, "Spatially engineered polarization states and optical vortices in uniaxial crystals," Opt. Express 18(10), 10848–10863 (2010).
- 19. I. Freund, "Second-harmonic generation of polarization singularities," Opt. Lett. 27(18), 1640–1642 (2002).
- S. Carrasco, B. E. A. Saleh, M. C. Teich, and J. Fourkas, "Second- and third-harmonic generation with vector Gaussian beams," J. Opt. Soc. Am. B 23(10), 2134–2141 (2006).
- A. Ohtsu, "Second-harmonic wave induced by vortex beams with radial and azimuthal polarizations," Opt. Commun. 283(20), 3831–3837 (2010).
- P. Stanislovaitis, A. Matijosius, M. Ivanov, and V. Smilgevicius, "Topological charge transformation of beams with embedded fractional phase step in theprocess of second harmonic generation," J. Opt. 19(10), 105603 (2017).
- P. Stanislovaitis, A. Matijošius, V. Šetkus, and V. Smilgevičius, "Peculiarities of second harmonic generation by paraxial beams with radial/azimuthal polarization in type II nonlinear crystal," Lith. J. Phys. 54(3), 142–149 (2014).
- P. Kurzynowski, W. A. Woźniak, M. Zdunek, and M. Borwińska, "Singularities of interference of three waves with different polarization states," Opt. Express 20(24), 26755–26765 (2012).
- T. A. Fadeyeva, C. N. Alexeyev, P. M. Anischenko, and A. V. Volyar, "Engineering of the space-variant linear polarization of vortex-beams in biaxially induced crystals," Appl. Opt. 51(10), C224–C230 (2012).
- L. Chen, W. Zhang, Q. Lu, and X. Lin, "Making and identifying optical superpositions of high orbital angular momenta," Phys. Rev. A 88(5), 053831 (2013).
- Z. Y. Zhou, D. S. Ding, Y. K. Jiang, Y. Li, S. Shi, X. S. Wang, and B. S. Shi, "Orbital angular momentum light frequency conversion and interference with quasi-phase matching crystals," Opt. Express 22(17), 20298–20310 (2014).

#### 1. Introduction

Vector beams [1-3], whose polarization can be parameterized with the projection of the Stokes parameters on the Poincaré sphere, have been widely concerned and applied in high numerical aperture focusing [4,5], laser machining [6], high capacity communication [7], optical manipulation [8,9] because of their interesting features. One particular example is the laser beams with different polarization states covering the entire Poincaré sphere, i.e., the socalled full Poincaré (FP) beams [10,11], hybrid vector beams [12–14]or polarization-singular beams [15]. Such light beams are constructed from the superpositions of two orthogonally polarized LG modes with different topological charges, which exhibit unique polarization singularities: points with a circular polarization (C-points), and lines with linear polarization (L-lines), on which the orientation and handedness of polarization ellipse are undefined, respectively [3,15]. The C-points and L-lines in various light fields, especially in the crystal, have been studied extensively [15–18]. For example, Cardano et al. presented the relationship between polarization singularities and topological charge of polarization-singular beams based on the analysis of C-points and L-lines [15].Lu et al. studied the evolution of polarization singularities when undergoing the Pockels effect [16]. Besides, Flossmann et al. revealed the polarization singularities that were unfolded through a birefringent crystal [17].

Also, recent years have witnessed a growing interest in the nonlinear optical process of second-harmonic generation (SHG) with vector beams [19–23]. Freund firstly provided a theoretical study on the SHG of the vector singularities of linearly polarized light and elliptically polarized light [19]. Besides, the SHG with a vector Gaussian beam [20] or the vortex beams in the sharp focusing regime [21] were investigated. In experiment, SHG with a fractional vortex beam of embedded fractional phase singularity [22] or with the radially and azimuthally polarized light in the paraxial regime [23] were conducted. However, we note that no experiment has been reported yet to connect the study of FP beams with the nonlinear SHG process. In addition to frequency conversion, the features of the topology of FP beams in the nonlinear optical regime have not yet been fully explored. Here we report such an experiment. The main point in our scheme is the employment of Type-II phase-matching KTP crystal, rather than the Type-I one. This is curtail for our experiment, as polarization

singularity in the FP beams is related to both horizontal and vertical polarization components, and type-I phase matching merely explores one-dimensional polarization. In contrast, the type-II configuration could convey the vectorial feature of polarization singularity into the intensity profile encoded by the SHG light fields.

## 2. Theory

Theoretically, an arbitrary-order FP beam is constructed from the superposition of a fundamental Gaussian mode and an arbitrary-order LG beam with orthogonal polarizations, namely [9,10],

$$\overline{E}(r,\varphi) = A \cdot LG_0^0(r,\varphi)\widehat{e}_1 + B \cdot LG_0^l(r,\varphi)\widehat{e}_2, \qquad (1)$$

where  $\hat{e}_1$  and  $\hat{e}_2$  are two unit vectors of left- and right-hand circular components, respectively, *A* and *B* are two controllable parameters used to regulate the profile of the FP beams. And the LG beams with p = 0 can be described as,

$$LG_{0}^{l}(r,\varphi) = E_{0}r^{l}L_{0}^{l}(\frac{2r}{w^{2}})\exp(-\frac{r^{2}}{w^{2}})\exp(il\varphi),$$
(2)

where  $\varphi$  is the azimuthal angle, w is the beam waist,  $L_0^l(\cdot)$  is the associated Laguerre polynomials, and l is an integer. As can be seen from Eq. (1), the FP beams consist of two controllable orthogonal polarized components, leading that the spatially varying polarization states at the transverse section. In contrast to the phase singularity, it describes the so-called spin-orbit state. Generally, these FP beams can be produced by interferometric methods, which is unstable [24], or via suitable birefringent crystal, which is inflexible [25]. In contrast, "q-plate" can stably and flexibly control the FP beams [15]. Thus here, we adopt a simplified "q-plate", i.e., the vortex half wave plate (VHWP) to generate FP beams. In order to adjust the parameters A and B, we need combine the half-wave plate (HWP) and the quarter-wave plate (QWP) to pretreat the polarization state. We assume the polarization state of the incident LG beams,  $LG_0^l$ , be horizontal. After passing through the HWP with its fast axis orienting at  $\alpha$  and the QWP with its fast axis orienting at 45°, the incident LG beams becomes

$$LG_0^l(r,\varphi) \begin{pmatrix} 1\\ 0 \end{pmatrix} \rightarrow \frac{(1-i)\sin(2\alpha)}{2} LG_0^l(r,\varphi) \begin{pmatrix} 1\\ i \end{pmatrix} + \frac{(1+i)\cos(2\alpha)}{2} LG_0^l(r,\varphi) \begin{pmatrix} 1\\ -i \end{pmatrix}, \quad (3)$$

where three column Jones matrices represent the horizontal polarization, right- and left-hand circular polarization, respectively (from left to right). And then, the polarization modulated LG beams are incident on the VHWP, which consists of liquid crystal polymers with half wave retardation and different molecular orientations along the azimuthal angle, and thus can be seen as a modified HWP with its fast axes  $\theta$  orienting at  $\theta = m\varphi/2$ , where  $\varphi$  is the azimuthal angle. Accordingly, its Jones matrix can be specified as

$$C_{VHWP} = \begin{pmatrix} \cos m\varphi & \sin m\varphi \\ \sin m\varphi & -\cos m\varphi \end{pmatrix}.$$
 (4)

Based on the Jones matrix theory, we can derive the light fields after passing through the VHWP as

$$LG_0^l(r,\varphi) \begin{pmatrix} 1\\ 0 \end{pmatrix} \rightarrow \frac{(1-i)\sin(2\alpha)}{2} LG_0^{l+m}(r,\varphi) \begin{pmatrix} 1\\ -i \end{pmatrix} + \frac{(1+i)\cos(2\alpha)}{2} LG_0^{l-m}(r,\varphi) \begin{pmatrix} 1\\ i \end{pmatrix}, (5)$$

Which revels clearly that the FP beams, mathematically expressed by Eq. (1), can be generated when l equals m. Here, by adjusting the fast axis orientation of HWP,  $\alpha$ , we can control the parameters A and B in the Eq. (1), thus producing a various types of FP beams.

In reality, the topological structures of polarization patterns embedded in the FP beams cannot be seen by a CCD camera that records only the intensity patterns. How to characterize these interesting polarization structures is therefore desirable. Here, we connect these interesting FP beams to the SHG effect, which enables the conversion from polarization patterns to intensity profiles. But if we merely employ the Type-I phase matching crystal to perform SHG, we can see that only one-dimensional polarization, e.g., the horizontal or vertical one, could participate the SHG process alone. Then the resultant SHG light field merely contains the information of one component polarization, and as such, the polarization topology will miss during SHG process. In contrast, if we employ type II phase matching crystal, we can see that both the horizontal and vertical polarization components are requested to accomplish the SHG cooperatively such that the polarization topology could be conveyed from invisible fundamental light to the visible SHG light. According to Eq. (5), we can deduce the horizontal and vertical components of the fundamental light as,

$$E_{H} = \frac{(1+i)\cos(2\alpha)}{2}LG_{0}^{0} + \frac{(1-i)\sin(2\alpha)}{2}LG_{0}^{l},$$
 (6a)

$$E_{V} = i \left[ \frac{(1+i)\cos(2\alpha)}{2} L G_{0}^{0} - \frac{(1-i)\sin(2\alpha)}{2} L G_{0}^{l} \right].$$
(6b)

Then under both the paraxial approximation and phase-matching condition, the SHG light field can be described by the following wave coupling equation,

$$\frac{dE_{SH}}{dz} = \frac{i\omega_{SH}^2 d_{eff}}{k_{SH}c^2} E_H E_V, \qquad (7)$$

where  $d_{eff}$  is the effective nonlinear coefficient,  $\omega$  and k are the angular frequency and wave vector of SHG beams, c is the velocity of light. And under the small-signal approximation, the SHG light field can be directly expressed as  $E_{SH} \propto E_H E_V$ , from which we can see that both horizontal and vertical polarizations are requested to contribute to the formation of the SHG light field. Thus, we can conclude that the trajectory of zero intensity in the SHG beams just correspond to horizontal and vertical polarizations appearing in the L-lines of FP beams, which is related to the topology of polarization singularities. In other words, type-II SHG provides a convenient method to reveal the polarization structure of FP beams, which just forms the compelling reason for us to perform such an experiment.

## 3. Experiment and analysis

We sketch the experimental setup in Fig. 1. The 50mW fundamental Gaussian mode of horizontal polarization is derived from a 1064nm laser source (Cobolt Rumba). After being collimated and expanded by a telescope, the light beam is incident on a spatial light modulation (SLM) (Hamamatsu, X10468-07). The SLM is a reflective device with a  $16mm \times 12mm$  window, consisting of  $792 \times 600$  pixels with a pixel pitch of  $20\mu m$ . Here it is used to display the LG modes via addressing the suitable holographic gratings, i.e., fork gratings, with intensity modulation [26]. By an optical 4f system and an adjustable iris, placed at its Fourier plane, we select out the 1storderdiffracted lights that carry the LG modes, reflected from SLM. And then, the generated LG modes pass through the half-wave plate (HWP), quarter-wave plate (QWP), and the vortex half wave plate (VHWP) (Thorlabs, WPV10L-1064 or WPV10-1064) to produce the desired FP beams via adjusting the fast axes

orienting of these three wave plates. Thus, the horizontal and vertical components of FP beams, serving as the fundamental waves, participate the SHG process. After filtering the 532*nm* SHG light beams through the filter, we use a color CCD camera (Thorlabs, DCU224C) to record the output intensity patterns.



Fig. 1. Experimental setup for SHG of FP beams in KTP, see the text for details.

## 3.1 Second harmonic generation of FP beam with l = 2

Here we perform the SHG of FP beam with order l = 2. We prepare and display the fork grating on the SLM, and we obtain the  $LG_0^1$  light beam by filtering the first-order diffraction with a 4f optical system and an iris placed in its Fourier plane. Here the VHWP with m = 1 is adopted. Firstly, we consider two particular cases, one is that the fast axis of HWP is adjusted to orientate at  $\alpha = 0^\circ$  while the other is  $\alpha = \pi/4$ . We know from Eq. (5) that the generated beams are purely a right-handed circularly polarized  $LG_0^0$  mode and a left-handed circularly polarized  $LG_0^2$  mode, respectively, as are shown by Fig. 2. As the states of polarization in these two cases are homogeneous, such that they are merely the results of SHG with scalar beams [27].



Fig. 2. Two trivial cases of SHG with scalar beams. Top panel:  $\alpha = 0^{\circ}$ . Bottom panel:  $\alpha = 45^{\circ}$ . (a1) and (b1): the intensity and polarization distributions of the fundamental beams. (a2) and (b2): numerical simulations of SHG light field's intensity patterns. (a3) and (b3): the experimental observations.



Fig. 3. The results for m = 1 with  $\alpha = 10^{\circ}, 20^{\circ}, 30^{\circ}, 40^{\circ}, 50^{\circ}, 60^{\circ}, 70^{\circ}$  and  $80^{\circ}$ . Top panel: transverse intensity and polarization distributions of fundamental FP beams, where the red circles indicate L-lines. Middle panel: numerical simulations for the transverse intensity patterns of SHG light field. Bottom panel: experimental observations.

In contrast, if  $\alpha$  takes other values, e.g.,  $10^\circ, 20^\circ, 30^\circ, 40^\circ, 50^\circ, 60^\circ, 70^\circ$  and  $80^\circ$ , the fundamental light can be prepared to acquire both the intensity and polarization profiles of FP beams. As shown in the top panel of Fig. 3, we numerically calculate the polarization distribution of FP beams at different orientations of  $\alpha$ . The Stokes parameters are used to determine the polarization pattern, as a consequence, the rectifying phase  $\chi$  calculated as  $\chi = \arcsin(s_3/s_0)/2$  and the orientation angle  $\psi$  of local polarization ellipses as  $\psi = \arg(s_1 + is_2)/2$ . One can see that all the "spiral" structure of polarization states possess the same topological index of T = 2, which is just equal to the topological charge of LG beams in Eq. (5). Those points satisfying  $\chi = 0$  just forms the trajectory of L-line, as was marked by the red circles in Fig. 3. The radius of L-lines decreases as  $\alpha$  is changing from 0° to 45°, and then increases as  $\alpha$  from 45° to 90°. The corresponding experimental observations of the SHG light fields are shown by the bottom panel of Fig. 3, from which we can verify that the dark cores appearing in the SHG beam are just corresponding to the horizontal and vertical polarizations. Also, the topological index of T = 2 is related to the two-fold rotational symmetry of the polarization pattern. Thus after SHG, we can see that the intensity profiles exhibit four-fold rotational symmetry. All the experimental observations in the bottom panels are obviously in good agreement with numerical simulations in the middle panel.

## 3.2Second harmonic generation of FP beam with l = 4

We further focus on a more complicated case with the fourth-order FP beams, which can be flexibly obtained by displaying the  $LG_0^2$  mode on the SLM and using the VHWP of m = 2. We can see that there is a four-fold rotational symmetry for input polarization patterns, which is related to the topological index of T = 4, in contrast to T = 2 for the case of m = 1. In order to compare the fundamental beams and the SHG beams intuitively, we calculate theoretically the polarization distribution of FP beams with the L-lines being marked by the red circles, as is shown in the top panel of Fig. 4. Accordingly, the SGH light field possesses the intensity profiles of eight-fold rotational symmetry, as a result of the interaction between topological structure of fundamental beams and type-II phase matching. We can see from both the middle and bottom panels that, for  $\alpha = 10^\circ$  the eight dark holes begin to appear on the boundary of the beam spot, and then it looks like a gear of eight rotation for  $\alpha = 30^\circ$ , and finally they

#### Research Article

#### Optics EXPRESS

almost merge into a whole dark core in the beam center for  $\alpha = 40^{\circ}$ , acquiring the doughnut shape of an OAM beam. However, as  $\alpha$  is increasing from 50° to 80°, we can see that it shows a reverse trend. Therefore our scheme of SHG of FP beams via type-II phase matching is able to provide a new way to visualize the topological structure hidden in the fundamental vectorial beams.



Fig. 4. The results for m = 2 with  $\alpha = 10^{\circ}, 20^{\circ}, 30^{\circ}, 40^{\circ}, 50^{\circ}, 60^{\circ}, 70^{\circ}$  and  $80^{\circ}$ . Top panel: transverse intensity and polarization distributions of fundamental FP beams. Middle panel: numerical simulations for the transverse intensity patterns of SHG light field. Bottom panel: experimental observations.

## 4. Conclusion

In summary, we conduct the type-II SHG with the FP beams. We adopt the combination of a commercial vortex half-wave plate (VHWP) and a phase only SLM to produce the desired FP beams in a robust way. The experimental observations with 2-order and 4-order FP beams serving as the fundamental light clearly reveal that the topological structures of polarization singularities could be effectively converted into the spatially varying intensity profiles of the visible SHG light fields under the type-II phase matching condition. Our work may provide a new perspective on the nonlinear interaction between structured light and nonlinear media.

## Funding

The National Natural Science Foundation of China (91636109, 11474238, 11604050, 11734011 and 61575041);The Fundamental Research Funds for the Central Universities at Xiamen University (20720160040);The Natural Science Foundation of Fujian Province of China for Distinguished Young Scientists (2015J06002);The program for New Century Excellent Talents in University of China (NCET-13-0495);National Key R&D Program of China (2017YFA0303700); Science and Technology Planning Project of Guangdong Province (2016B010113004).